

## Bachelor of Arts 3<sup>rd</sup> Semester

### Mathematics (Differential Equations - 1)

Time allowed: 3 Hours

Max. Marks: 30

**Note:** Attempt five questions in all selecting at least two questions from each section. All questions carry equal marks.

**I.**

#### UNIT-I

- (a) If  $\frac{1}{N} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$  is function of  $x$  only, say  $f(x)$ , then show that  $e^{\int f(x)dx}$  is an integrating factor of  $Mdx + Ndy = 0$ .

- (b) Given that the differential equation (3,3)  
 $(2x^2y^2 + y)dx - (x^3y - 3x)dy = 0$  has an integrating factor of the form  $x^h y^k$ , find its general solution.

**II.**

- (a) Solve:

$$xyp^2 - (x^2 + y^2 - 1)p + xy = 0.$$

- (b) Solve the differential equation: (3,3)

$$p = \tan \left( x - \frac{p}{1 + p^2} \right)$$

**III.**

- (a) Obtain the primitive and singular solution, if it exists of the equation,

$$xp^2 - 2yp + 4x = 0.$$

- (b) Find the orthogonal trajectories of the family of coaxial circles  $x^2 + y^2 + 2gx + c = 0$ , where  $g$  is the parameter. (3,3)

**IV.**

- (a) Solve the differential equation:

$$\frac{d^2y}{dx^2} + a^2y = \sec ax.$$

- (b) Solve: (3,3)

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x.$$

#### UNIT-II

**V.**

- (a) Solve the differential equation:

$$x^2 \frac{d^3y}{dx^3} + 3x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{y}{x} = \log x.$$

- (b) Solve: (3,3)

$$(x+3)^2 \frac{d^2y}{dx^2} - 4(x+3) \frac{dy}{dx} + 6y = x.$$

**VI.** (a) Solved by the method of variation of parameters:

$$(D^2 + 3D + 2)y = \sin e^x$$

(b) solve  $\frac{d^2y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = e^x \sec$  by the method of removal of first derivative,

**VII.** (a) Solve:

$$x^2 \frac{d^2y}{dx^2} - (x^2 + 2x) \frac{dy}{dx} + (x + 2)y = x^3 e^x$$

by changing the dependent variable.

(b) Transform the differential equation:

$$(\cos x)y'' + (\sin x)y' - 2y \cos^3 x = 2 \cos^5 x$$

(3,3)

Into the one having as independent variable where  $z = \sin x$  and solve it.

**VIII.** (a)

$$\frac{dx}{dt} + \frac{dy}{dt} = 3x - 6y$$

$$\frac{dx}{dt} - \frac{dy}{dt} + x + 4y = 0$$

(b) Solve:  $D^2X - 3Dy + 4x = 0$  and  $3Dx + D^2y + 4y = 0$  where  $D = \frac{d}{dt}$

(3,3)